Chapter 4 Targets

0a) $\frac{2+3i}{1-2i}\cdot\frac{(1+2i)}{(1+2i)}$

$= \frac{2+4i+3i+6i^2}{1-4i^2}$

$\frac{-4+7i}{5}$

$\boxed{-\frac{4}{5}+\frac{7}{5}i}$

0b) $i^{37} = (i^2)^{18}\cdot i$

$= (-1)^{18} i$

$= \boxed{i}$

1a) yes; no; no

1b) 4; 5

1c) $3(1)-5=0$?

$-2=0$? no!

1d) $(-5)^3+2(-5)^2-15(-5)=0$?

$-125+50+75=0$

$0=0$ ✓

$\boxed{\text{Yes}}$

1e) $(x+2i)(x-2i)(x-3)(x+3)$

$(x^2-4i^2)(x^2-9)$

$(x^2+4)(x^2-9)$

$x^4-9x^2+4x^2-36$ FOIL!

$\boxed{x^4-5x^2-36}$

2a) $b^2-4ac = (-4)^2-4(4)(-15)$

$= 16+240$

$= 256 \rightarrow 2$ real roots

2b) $b^2-4ac = (2)^2-4(3)(5)$

$= -56 \rightarrow 2$ imaginary roots!

2c) $\frac{-4x^2}{-4}-\frac{11x}{-4}=\frac{7}{-4}$

$x^2+\frac{11}{4}x=-\frac{7}{4}$ $\left(\frac{b}{2}\right)^2$

$x^2+\frac{11}{4}x+\frac{121}{64}=-\frac{7}{4}+\frac{121}{64}$

$\sqrt{\left(x+\frac{11}{8}\right)^2}=\sqrt{\frac{9}{64}}$

$\left(x+\frac{11}{8}\right)=\pm\frac{3}{8}$

$\boxed{x=-1,\ -\frac{7}{4}}$

2d) $b^2-4ac=(-14)^2-4(5)(11)$

$=-24$

$x=\frac{14\pm\sqrt{-24}}{2(5)}$ $\begin{matrix}\sqrt{6}\\ \sqrt{4}\end{matrix}$

$=\frac{14\pm 2i\sqrt{6}}{10}$

$$\boxed{x=\frac{7\pm i\sqrt{6}}{5}}$$

3a) $x+3 \overline{)\,2x^3+3x^2-8x+3}$

$2x^2-3x+1$

$-2x^3+6x^2$

$-3x^2-8x$

$+3x^2+9x$

$x+3$

$-x+3$

0

$$\boxed{2x^2-3x+1}$$

3b) $-5 \overline{)\,3 \quad 4 \quad -12}$

$-15 \quad 55$

$3 \quad -11 \quad \boxed{43}$

$$\boxed{3x-11 \text{ R } 43}$$

ii) $-1 \overline{)\,1 \quad 0 \quad -3 \quad 0 \quad 12}$

$-1 \quad 1 \quad 2 \quad -2$

$1 \quad -1 \quad -2 \quad 2 \quad 10$

$$\boxed{x^3-x^2-2x+2 \text{ R } 10}$$

3c) $-1 \overline{)\,2 \quad 4 \quad -1 \quad 0 \quad 9}$

$-2 \quad -2 \quad 3 \quad -3$

$2 \quad 2 \quad -3 \quad 3 \quad \boxed{6}$

Remainder $=6 \therefore (x+1)$ is not a factor!

3d) $0=(2)^4+k(2)^3-14(2)^2$

$0=16+8k-56$

$40=8k$

$$\boxed{5=k}$$

3e) $2 \overline{)\,2 \quad -7 \quad 4 \quad 4}$

$\downarrow \quad 4 \quad -6 \quad -4$

$2 \overline{)\,2 \quad -3 \quad -2 \quad 0}$

$4 \quad 2$

$2 \quad 1 \quad 0$

$2x+1$

$$\boxed{(x-2)^2(2x+1)}$$

4a) $x^3+3x^2-6x-8=0$

$\frac{p}{q}: \frac{\pm(1,2,4,8)}{\pm 1}$

complex roots: $\pm\frac{1}{2}, \pm\frac{1}{3}$

4b) check:

$f(-4)=0$ ✓

$f(-2)=0$ ✓

$f(2)=0$ ✓

4c) rational roots: $-4, -2, 2$

4d) $f(x)=x^3-2x^2-19x+20$

$\frac{p}{q}: \pm(1,2,4,5,10,20)$

roots: $-4, 1, 5$

$f(x)=36x^4-13x^2+1$

$\frac{p}{q}: \frac{\pm 1}{(1,2,3,4,6,9,12,18,36)}$

zero on calc!

$\frac{1}{3}$ | 36 0 -13 0 1

↓ 12 4 -3 -1

$\frac{1}{2}$ | 36 12 -9 -3 | 0

18 15 3

$-\frac{1}{2}$ | 36 30 6 | 0

-18 -6

36 12 | 0 →

6a) $\frac{3x}{2x+1} - \frac{4}{2x-1} = 1$ $(2x+1)(2x-1)$

$3x(2x-1) - 4(2x+1) = (2x+1)(2x-1)$

$6x^2-3x-8x-4 = 4x^2-1$

$6x^2-11x-4 = 4x^2-1$

$2x^2-11x-3=0$

$x = \frac{11 \pm \sqrt{(-11)^2-4(2)(-3)}}{2(2)}$

$\boxed{x = \frac{11 \pm \sqrt{145}}{4}}$ $\quad x \neq \frac{1}{2}, -\frac{1}{2}$

6b) $\frac{-3x-29}{(x-7)(x+3)} = \frac{A}{x-7} + \frac{B}{x+3}$ $(x-7)(x+3)$

$-3x-29 = A(x+3) + B(x-7)$

$A=-5, B=2$

$= \frac{2}{x+3} + \frac{-5}{x-7}$

6c) $\left(\frac{2x}{4} - \frac{5x+1}{3} > 3\right)12$

$6x-4(5x+1)=36$

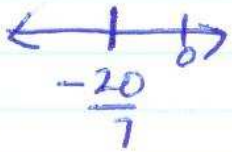
$6x-20x-4=36$

$-14x=40$

$x=\frac{-20}{7}$

$\boxed{x<-\frac{20}{7}}$

try 0: $0-0>12$ ✗



7a) $(\sqrt{x-2})^2 = (6)^2$

$x-2=36$

$\boxed{x=38}$

$(\sqrt{6x+12})^2 = (1+\sqrt{4x+9})^2$ $(1+\sqrt{4x+9})$

$6x+12 = 1+2\sqrt{4x+9}+4x+9$

$6x+12 = 10+4x+2\sqrt{4x+9}$

$\frac{2x+10}{2} = \frac{2\sqrt{4x+9}}{2}$

$(x+5)^2 = (\sqrt{4x+9})^2$

$x^2+10x+25 = 4x+9$

$x^2+6x+16=0$

$x = \frac{-6\pm\sqrt{(-6)^2-4(1)(16)}}{2}$

$= \frac{-6\pm\sqrt{-28}}{2}$

$= \frac{-6\pm 2i\sqrt{7}}{2}$

$\boxed{x=-3\pm i\sqrt{7}}$

7b) $\sqrt{3r+5} > 1$

$(\sqrt{3r+5})^2 = (1)^2$

$3r+5=1$

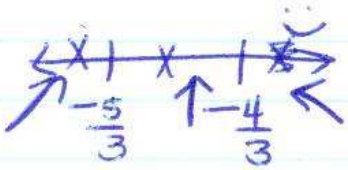
$3r=-4$

$r=-\frac{4}{3}$

impt values: $\frac{-5}{3}, \frac{-4}{3}$

$3r+5=0$

$3r=-5$



$\boxed{r > -\frac{4}{3}}$

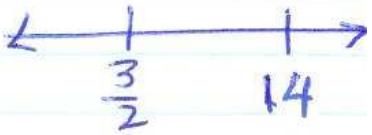
$\sqrt{2t-3} < 5$ impt values: $\frac{3}{2}, 14$

$(\sqrt{2t-3})^2 = (5)^2$

$2t-3=25$

$2t=28$

$t=14$



try 0: $\sqrt{-3} < 5$ X

try 2: $\sqrt{1} < 5$ ✓

try 15: $\sqrt{27} < 5$ X

$\boxed{-\frac{3}{2} < t < 14}$