Lake Zurich High School

Mathematics Department

GETTING STARTED PACKET

RESOURCE COMPANION

An Algebra Review

Find the distance between (4, -5) and (8, 3)

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(8-4)^2 + (3-(-5))^2}$$
$$d = \sqrt{(4)^2 + (8)^2}$$
$$d = \sqrt{16 + 64}$$
$$d = \sqrt{80}$$
$$d = \sqrt{2^4 \bullet 5}$$
$$d = 2^2\sqrt{5}$$
$$d = 4\sqrt{5}$$

Find the slope and the midpoint for the two points (5, -1) and (-3, 4)

Slope Formula: $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$

$$m = \frac{(4-(-1))}{(-3-5)}$$
$$m = \frac{5}{-8}$$
$$m = -\frac{5}{8}$$

Midpoint Formula: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

$$\left(\frac{5+(-3)}{2}, \frac{-1+4}{2}\right)$$
$$\left(\frac{2}{2}, \frac{3}{2}\right)$$
$$\left(1, \frac{3}{2}\right)$$

Laws of Exponents:

multiplication	**negative exponents**	**division**
$x^m \cdot x^n = x^{n+m}$	$x^{-n} = \dfrac{1}{x^n}$	$\dfrac{x^n}{x^m} = x^{n-m}$
$x^5 \cdot x^7 = x^{12}$	$x^{-4} = \dfrac{1}{x^4}$	$\dfrac{x^8}{x^3} = x^5$

ex1) $\dfrac{a^3b^{-5}c^{-2}d^7}{a^8b^3c^{-8}e^{-3}} = \dfrac{c^{-2-(-8)}d^7e^{0-(-3)}}{a^{8-3}b^{3-(-5)}} = \dfrac{c^6d^7e^3}{a^5b^2}$

ex2) $\left(\dfrac{x^3}{y^4}\right)^{-2} = \dfrac{x^{-6}}{y^{-8}} = \dfrac{y^8}{x^6}$

Write the equation of the line with a slope of -2 and passing through (4, 7).

Use the point slope form: $y - y_1 = m(x - x_1)$

$$y - 7 = -2(x - 4)$$

$$y - 7 = -2x + 8$$

$$\boxed{y = -2x + 15}$$

Write the equation of the line passing through (4,5) and (7,9)

$$m = \frac{(9-5)}{(7-4)} = \frac{4}{3}$$

$$y - 5 = \frac{4}{3}(x - 4)$$

$$y - 5 = \frac{4}{3}x - \frac{16}{3}$$

$$y = \frac{4}{3}x - \frac{16}{3} + \frac{15}{3}$$

$$\boxed{y = \frac{4}{3}x - \frac{1}{3}}$$

Write the equation of the line parallel to 3x + 5y = 7 passing through (-1, 2)

First, find the slope of the line.

$$5y = -3x + 7$$

$$y = \frac{-3}{5}x + \frac{7}{5}$$

slope: $m = \frac{-3}{5}$

Two lines that are parallel have the same slope.

$$y - 2 = \frac{-3}{5}(x - (-1))$$

$$y - 2 = \frac{-3}{5}(x + 1)$$

$$y - 2 = \frac{-3}{5}x + \frac{-3}{5}$$

$$y = \frac{-3}{5}x + \frac{-3}{5} + \frac{10}{5}$$

$$\boxed{y = \frac{-3}{5}x + \frac{7}{5}}$$

Slope intercept form

If we were to convert this linear equation in slope intercept form to **standard form** we would multiply each side by 5 and then add 3x to both sides so that it would be →

$$\boxed{3x + 5y = 7}$$

Standard form

Write the equation of the line perpendicular to 4x + 3y = 9 passing through (7, -3)

First, find the slope of the line.

$$3y = -4x + 9$$

$$y = \frac{-4}{3}x + 3$$

slope: $m = \frac{-4}{3}$

Two lines that are perpendicular have slopes that are opposite reciprocals of each other.

$$m = \frac{-4}{3} \rightarrow \perp m = \frac{3}{4}$$

$$y - (-3) = \frac{3}{4}(x - 7)$$

$$y + 3 = \frac{3}{4}x - \frac{21}{4}$$

$$y = \frac{3}{4}x - \frac{21}{4} - \frac{12}{4}$$

$$\boxed{y = \frac{3}{4}x - \frac{33}{4}}$$

Solve the following quadratic equations by factoring:

$12x^2 - 7x - 10 = 0$

$(3x + 2)(4x - 5) = 0$

$3x + 2 = 0$ or $4x - 5 = 0$

$3x = -2$ $\quad$ $4x = 5$

$$\boxed{x = \frac{-2}{3} \qquad x = \frac{5}{4}}$$

$5x^2 + 28x - 32 = 0$

$(5x - 4)(3x + 8) = 0$

$5x - 4 = 0$ or $3x + 8 = 0$

$5x = 4$ $\quad$ $3x = -8$

$$\boxed{x = \frac{4}{5} \qquad x = \frac{-8}{3}}$$

Solve the following quadratic equation by using square roots:

$x^2 + 5 = 11$

$x^2 = 6$

$$\boxed{x = \pm\sqrt{6}}$$

$x^2 - 8 = 32$

$x^2 = 40$

$$\boxed{x = \pm 2\sqrt{10}}$$

Solve the following quadratic equation using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ex1. $3x^2 + 4x - 2 = 0$

a = 3
b = 4
c = -2

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \bullet 3 \bullet (-2)}}{2 \bullet 3}$$
$$x = \frac{-4 \pm \sqrt{16 + 24}}{6}$$
$$x = \frac{-4 \pm \sqrt{40}}{6}$$
$$x = \frac{-4 \pm 2\sqrt{10}}{6}$$
$$x = \frac{2\left(-2 \pm \sqrt{10}\right)}{6}$$
$$\boxed{x = \frac{-2 \pm \sqrt{10}}{3}}$$

ex2. $5x^2 - 12x + 3 = 0$

a = 5
b = -12
c = 3

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4 \bullet 5 \bullet 3}}{2 \bullet 5}$$
$$x = \frac{12 \pm \sqrt{144 - 60}}{10}$$
$$x = \frac{12 \pm \sqrt{84}}{10}$$
$$x = \frac{12 \pm 2\sqrt{21}}{10}$$
$$x = \frac{2\left(6 \pm \sqrt{21}\right)}{10}$$
$$\boxed{x = \frac{6 \pm \sqrt{21}}{5}}$$

ex3. $6x^2 + 2x + 5 = 0$

a = 6
b = 2
c = 5

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \bullet 6 \bullet 5}}{2 \bullet 6}$$
$$x = \frac{-2 \pm \sqrt{4 - 120}}{12}$$
$$x = \frac{-2 \pm \sqrt{-116}}{12}$$

Because a negative is under the radical, there is **no real solution**.

Function notation Given: $f(x) = 3x^2 + 2x - 5$

Find: f(2)
$f(2) = 3(2)^2 + 2(2) - 5$
$f(2) = 3 \bullet 4 + 4 - 5$
$f(2) = 12 + 4 - 5$
$f(2) = 11$

Find: f(-3)
$f(-3) = 3(-3)^2 + 2(-3) - 5$
$f(-3) = 3(9) - 6 - 5$
$f(-3) = 27 - 6 - 5$
$f(-3) = 16$

Multiplying Polynomials

ex1) $(2x + 3)(6x - 5)$

$(2x)(6x) + (2x)(-5) + (3)(6x) + (3)(-5)$
$12x^2 - 10x + 18x - 15$
$12x^2 + 8x - 15$

ex2) $(3x + 4)(5x^2 - 7x + 6)$

$(3x)(5x^2) + (3x)(-7x) + (3x)(6) + (4)(5x^2) + (4)(-7x) + (4)(6)$
$15x^3 - 21x^2 + 18x + 20x^2 - 28x + 24$
$15x^3 - x^2 - 10x + 24$

ex3) $(x + y)^2$
$(x + y)(x + y)$
$x^2 + xy + xy + y^2$
$x^2 + 2xy + y^2$ fast rule → square the first term → x^2
double the product of the to terms → $2xy$
square the last term → y^2

we need to be using he fast rule when squaring a binomial ! ! !

ex4) $(3x - 5)^2$
fast rule → square the first term → $(3x)^2$ → $9x^2$
double the product of the to terms → $2(3x)(-5)$ → $-30x$
square the last term → $(-5)^2$ → 25

put it together → $(3x - 5)^2 = 9x^2 - 30x + 25$

Algebra II students need to use the fast rule really FAST!

Don't forget the middle term!

ex5) $(2x + 3y)^2$

fast rule → $4x^2 + 12xy + 9y^2$

ex6) $(7x - 3)^2$

fast rule → $49x^2 - 42x + 9$

ex7) $(5x - 7)(5x + 7)$

$(5x)(5x) + (5x)(7) + (-7)(5x) + (-7)(7)$
$25x^2 + 35x - 35x - 49$ notice the middle terms cancel
$15x^2 - 49$

Solve the following system by the substitution method.

$2x + 3y = 7$
$x + 2y = 5$

Solve for x in the second equation $x = -2y + 5$

Now, substitute this value of x into the first equation.

$2(-2y + 5) + 3y = 7$
$-4y + 10 + 3y = 7$
$-y = -3$
$y = 3$ now substitute this into $x = -2y + 5$

$x = -2(3) + 5$
$x = -6 + 5$
$x = -1$

The solution is **(-1, 3)**

Solve the following system by the elimination method (linear combinations).

$3x + 4y = -8$
$5x – 3y = 35$

First, decide on a variable to eliminate.
We chose to eliminate y because of the different signs on their coefficients.

Multiply the 1st equation by 3 → $9x + 12y = -24$
Multiply the 2^{nd} equation by 4 → $20x – 12y = 140$

Add the two equations $29x = 116$

Solve for x $x = 4$

Now substitute x = 4 into one of the original equations to solve for y.

$3(4) + 4y = -8$
$12 + 4y = -8$
$4y = -20$
$y = -5$ → → → → → so the solution is the point **(4, -5)**

Graphing Parabolas

Determine the vertex, the x-intercepts, the y-intercepts and sketch the graph of the following parabola.

$$y = 2x^2 + x - 6$$

axis of symmetry: $x = \frac{-b}{2a}$

a = 2
b = 1

$$x = \frac{-1}{2 \bullet 2}$$

$$x = \frac{-1}{4}$$

now substitute that x value

into the equation to find y.

$$y = 2\left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{4}\right) - 6$$

$$y = 2\left(\frac{1}{16}\right) + \left(\frac{-1}{4}\right) - 6$$

$$y = \frac{1}{8} + \frac{-1}{4} - 6$$

$$y = \frac{1}{8} + \frac{-2}{8} + \frac{-48}{8}$$

$$y = \frac{-49}{8}$$

vertex is at $\left(\frac{-1}{4}, \frac{-49}{8}\right)$

x–intercepts → set y = 0 and solve for x

$0 = 2x^2 + x - 6$
$0 = (2x - 3)(x + 2)$

$2x - 3 = 0 \qquad x + 2 = 0$

$$x = \frac{3}{2} \qquad x = -2$$

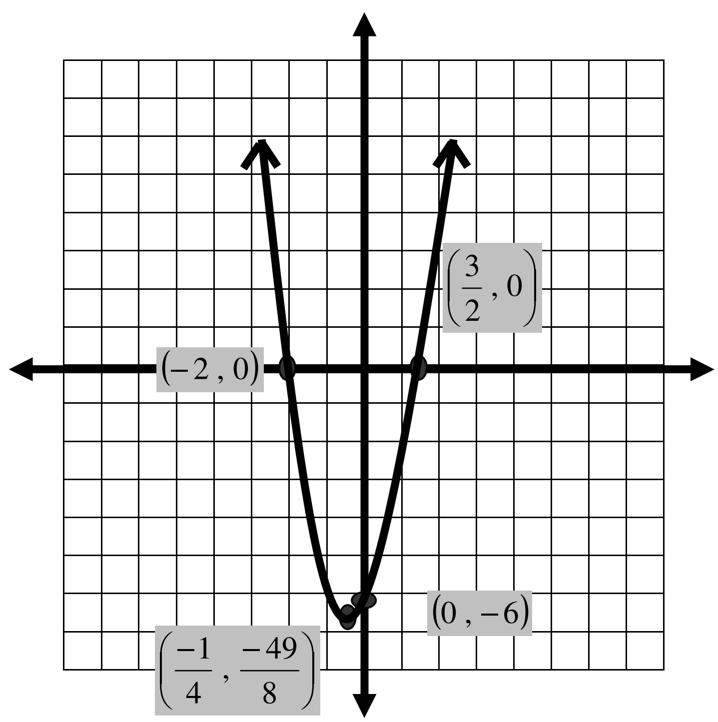
we have two x-intercepts: $\left(\frac{3}{2}, 0\right)$ *and* $(-2, 0)$

y–intercepts → set x = 0 and solve for y

$y = 2(0)^2 + 0 - 6$

y = -6

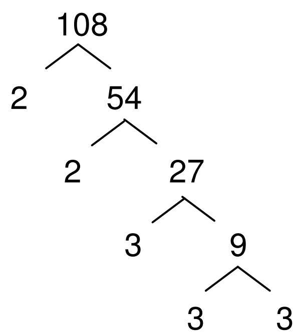
we have a y-intercept at: (0, -6)



Radicals:

ex1) $\sqrt{108}$

Factor tree for prime factorization



$108 = 2^2 \bullet 3^3$

$\sqrt{108}$

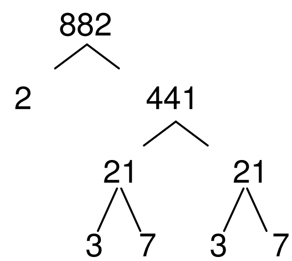
$\sqrt{2^2 \bullet 3^3}$

$2 \bullet 3\sqrt{3^1}$

$6\sqrt{3}$

ex2) $\sqrt{882}$

factor tree for prime factorization



$882 = 2 \bullet 3^2 \bullet 7^2$

$\sqrt{882}$

$\sqrt{2 \bullet 3^2 \bullet 7^2}$

$3 \bullet 7\sqrt{2^1}$

$21\sqrt{2}$

ex3) $\sqrt{x^5\ y^7\ z^6}$

$\sqrt{x^2\ x^2\ x \cdot y^2\ y^2\ y^2\ y \cdot z^2\ z^2\ z^2}$

$x \cdot x \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z\sqrt{x \bullet y}$

$x^2 y^3 z^3 \sqrt{xy}$

ex4) $3\sqrt{5} \bullet 4\sqrt{7}$

$3 \cdot 4\sqrt{5 \cdot 7}$

$12\sqrt{35}$

ex5) $3x\sqrt{2x} \bullet 4x^2\sqrt{5x}$

$3x \cdot 4x^2\sqrt{2x \cdot 5x}$

$12x^3\sqrt{2 \cdot 5x^2}$

$12x^3 \cdot x\sqrt{2 \cdot 5}$

$12x^4\sqrt{10}$

ex6) $5\sqrt{3} + 4\sqrt{2} + 7\sqrt{3}$

$12\sqrt{3} + 4\sqrt{2}$

ex7) $\sqrt{\frac{2}{3}}$

$\frac{\sqrt{2}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}}$

ex8) $\sqrt{\frac{24}{15}}$

$\sqrt{\frac{8}{5}} = \frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{2}}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{10}}{5}$

Simplifying Rational Expressions

Step 1: Factor the numerator and the denominator
Step 2: Cancel the common factors

ex1) $\dfrac{3x^4 + 12x^2}{15x^3}$

$\dfrac{3x^2(x^2 + 4)}{15x^3}$

$\dfrac{(x^2 + 4)}{5x}$

ex2) $\dfrac{x^2 + 5x + 6}{x^2 - 4}$

$\dfrac{(x + 2)(x + 3)}{(x-2)(x+2)}$

$\dfrac{(x + 3)}{(x-2)}$

ex3) $\dfrac{5x^2 - 5x - 60}{x^2 - 2x - 8}$

$\dfrac{5(x^2 - x - 12)}{x^2 - 4x + 12}$

$\dfrac{5(x - 4)(x + 3)}{(x - 4)(x + 2)}$

$\dfrac{5(x + 3)}{(x + 2)}$

Solving rational equations:

ex1) $\dfrac{5}{3} + \dfrac{7}{2x} = \dfrac{1}{x}$

$6x \bullet \left(\dfrac{5}{3} + \dfrac{7}{2x}\right) = \dfrac{1}{x} \bullet 6x$

$10x + 21 = 6$

$10x = -15$

$x = -\dfrac{3}{2}$

ex2) $\dfrac{3}{7} = \dfrac{2x-3}{3x+4}$

$7(3x+4)\left(\dfrac{3}{7}\right) = \left(\dfrac{2x-3}{3x+4}\right)7(3x+4)$

$(3x+4)(3) = (2x-3)7$

$9x + 12 = 14x - 21$

$33 = 5x$

$\dfrac{33}{5} = x$

Solving absolute value equations:

ex1) $|3x + 5| = 9$

break into two equations

$3x + 5 = 9$ or $3x + 5 = -9$

$3x = 4$ $\quad$ $3x = -14$

$x = \dfrac{4}{3}$ $\quad$ $x = \dfrac{-14}{3}$

ex2) $|2x - 7| = 18$

break into two equations

$2x - 7 = 18$ or $2x - 7 = -18$

$2x = 25$ $\quad$ $2x = -11$

$x = \dfrac{25}{2}$ $\quad$ $x = \dfrac{-11}{2}$

Absolute value inequalities

Ex1) $|2x-6| \geq 8$

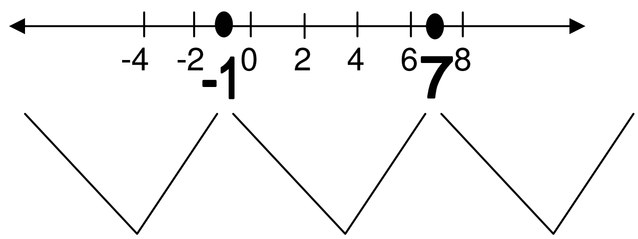
Forget about the inequality for now and just consider

$$|2x-6| = 8$$

break into two equations

$$\begin{array}{ll} 2x-6=8 & \quad 2x-6=-8 \\ 2x=14 & \quad 2x=-2 \\ x=7 & \quad x=-1 \end{array}$$

plot the zeros on the number line



test a point in each interval to find the solution set

testing <u>-4</u> in the left interval → $|2(-4)-6| = |-8-6| = |-14| = 14$ which is ≥ 8 TRUE

testing <u>0</u> in the middle interval → $|2(0)-6| = |0-6| = |-6| = 6$ which is not ≥ 8 FALSE

testing <u>8</u> in the right interval → $|2(8)-6| = |16-6| = |10| = 10$ which is ≥ 8 TRUE

Therefore, the solution is the left and right intervals.

$$\{x : x \leq -1 \text{ or } x \geq 7\}$$

an ALTERNATIVE notation

$$(-\infty, -1] \cup [7, +\infty)$$

NOTE: This approach on this inequality may be different from the technique that you learned in Algebra I. It is the approach that will be used in H-Algebra II. Don't worry. It is much easier than it looks on this page. It **<u>will</u>** be reviewed in H-Algebra II.